Singular Soliton to the Hyperbolic Generalization of the Burgers Model

Haci Mehmet Baskonus[1], Faruk Düşünceli[2]

1. Department Computer Engineering, University of Munzur, TURKEY, Tunceli,
2. Faculty of Architecture and Engineering, Mardin Artuklu University, Mardin, Turkey

hmbaskonus@gmail.com, farukdusunceli@artuklu.edu.tr

Abstract* – *In this paper, new singular soliton solution is found to the hyperbolic generalization of the Burgers equation. 2D and 3D graphs are also presented. At the end of paper, a conclusion is introduced as well by mentioning novel aspects of paper.

Keywords – Exponential function method, Hyperbolic Generalization of the Burgers equation, Singular soliton solution.

Introduction

The models arising in wave propagation are mostly expressed in time domain [1-4]. Such models also introduces the shapes of waves from time to time. One of such model is hyperbolic generalized Burgers equation (GBE) defined by [5-7]

$$\tau u_{tt} + u_t + uu_x + Bu_x - \kappa u_{xx} - \lambda u(u-s)(u+q) = 0, \quad (1)$$

where $\tau, B, \kappa, \lambda, s, q$ are arbitrary real constants.

General Properties of EFM

General and more deeper propertiess of EMF have been presented by varios scientist to the literature in [8,9].

Implementation of the EFM

Applying $u(x,t) = U(\xi),\ \xi = kx - ct$ into Eq.(1), we get following nonlinear ordinary differential equation

$$\left(\tau c^2 - \kappa k^2\right)U'' + (kB - c)U' + kUU' - \lambda U^3 + (s\lambda - q\lambda)U^2 + s\lambda q U = 0, \ (2)$$

Considerin balance principle, the value of N can be found as

$$N = 1. (3)$$

Then, we can write following equations;

$$U = A_0 + A_1 \exp(-\Omega(\xi)), (4)$$

$$U' = A_1 \exp(-\Omega(\xi))\Omega', (5)$$

$$U'' = \cdots (6)$$

where $A_1 \neq 0$. Using Eqs.(4,5,6) into Eq.(2), we get an algebraic system of equations. Solving this system of equations, we reach following coefficients

$$A_0 = \frac{1}{2}(-q + wA_1),\ s = -\frac{2c - 2Bk + kq + q\lambda A_1}{2\lambda A_1},\ \tau = \frac{2k^2\kappa + (k + \lambda A_1)A_1}{2c^2}, \mu = \frac{1}{4}\left(w^2 - \frac{q^2}{A_1^2}\right).$$

Substituting these coefficients into Eq.(4), we find the following singular soliton solution as

$$u(x,t) = \frac{-q + wA_1}{2} + \frac{\left(q^2 - w^2A_1^2\right)\coth(k\varpi x - \varpi ct + \varpi E)}{2q + 2wA_1 \coth(k\varpi x - \varpi ct + \varpi E)}, (7)$$

where $q^2/A_1^2 > 0$ *and* $\varpi = q/2A_1$ for validity condition of Eq.(7).

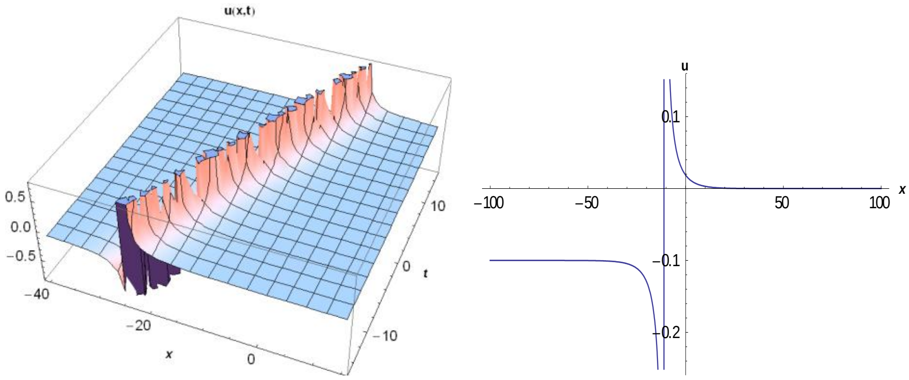

Fig.1 The 3D and 2D surfaces of Eq.(7)

Conclusion

In this paper, we have applied EFM to the GBE. singular soliton solution has also obtained. It can be observed that this solution has satisfied to the GBE via various computational programs. For better understanding of physical meanings of results, 2D and 3D surfaces have also plotted. Therefore, it can be said that this method is very powerfull and can be applied other nonlinear with high nonlinearity.

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