Analysis of Parameter Sensitivity to Enhance the Detection of Leaks in Sealed Landfills

Marco Vocciante [1], Vincenzo Dovì [2]

1. DCCI, University of Genova, ITALY, Genova, Via Dodecaneso 31, E-mail: marco.vocciante@unige.it

2. VD Consulting Ltd, UK, London, 124 City Road, Email: vgdovi@gmail.com

***Abstract* – It was shown in a previous article that the presence of leaks in sealed landfills due to confinement failures can be assessed measuring surface moisture and relating it to leaks at the bottom of the landifll through a regularised inversion algorithm based on Richard's equation with a piecewise linear boundary condition. Under the assumption of absence of leaks as the null hypothesis, the algorithm provides the value of the relevant F-statistic as a function of the accuracy of soil moisture measurements and of physical and meteorological parameters.**

In this presentation we take into account the uncertainties of the parameters and estimate the corresponding ranges of the F-values by evaluating the derivatives of the F-statistic with respect to the parameters.

Keywords – Detection of confinement failures, Richards' Equation, Inverse problem, Regularisation techniques, Sensitivity analysis of F-statistic

Introduction

Waste management relies worldwide on landfill disposal for over two thirds of the waste generated [1] either through some form of landifill management or open dumping. Low-income countries presently using open dumping are expected to extend the use of controlled landfills, which may prove to be locally more appropriate solutions with respect to other advanced solutions such as recycling, composting, and incineration. Hence, the increasing role of sealed landfills and the necessity of a better modeling strategy for their monitoring and control.

The presence of leaks in landfill liners is generally monitored using wells below the water table and sensors in the vadose zone [2]. In addition to instrument malfunctioning of both sensors and wells (which can give rise to data misinterpretation), sensors in the vadose zone may fail to detect a narrow plume close to the liner [3], whereas sensors located in deeper regions could fail to detect the plume due to an impeding layer that diverts percolating water horizontally [4]. It was shown in a previous article [5] that the presence of leaks in sealed landfills due to confinement failures can be assessed measuring surface moisture and relating it to leaks at the bottom of the landifll through a regularised inversion algorithm based on Richard's equation with a piecewise linear boundary condition. Under the assumption of absence of leaks as the null hypothesis, the algorithm provides the value of the relevant F-statistic as a function of the accuracy of soil moisture measurements and of physical and meteorological parameters.

In this paper we discuss how the uncertainties of the parameters can affect the value of the F-statistic and consequently the reliability of the model in the assessment of leaks. The paper is organized as follows: in the next section a previously developed inverse model will be presented. The sensitivity analysis which is the main goal of this paper will be consider in the following section, whereas the significance of the algorithm will be examined in the Conclusions.

Percolation model

The one-dimensional pressure-head form of Richards' equation is given by [6]

$$\frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right]-S(z)=C(\psi)\frac{\partial \psi}{\partial t} \qquad (1)$$

where ψ is the pressure head, $K(\psi)$ is the hydraulic conductivity $C(\psi)=\dfrac{d\theta}{d\psi}$ is the differential water capacity and θ is the volumetric water content, S is water uptake (which is negative when considering landfills because it gives the water generated by wastes) and z is the vertical coordinate pointing upward.

The initial condition $\psi_0(z)$ and boundary conditions at z=0 (the bottom of the landfill) and at z=L (at the surface of the landfill) are to be provided.

The flux at the surface $\left[K(\psi)\left(\dfrac{\partial\psi}{\partial z}+1\right)\right]_{z=L}$ is equal to the amount of water released by evapotranspiration per unit surface minus the rate of rainfall infiltration amount of rain, i.e. $\left[K(\psi)\left(\dfrac{\partial\psi}{\partial z}+1\right)\right]_{z=L}=q_0(t)-q_1(t)$.

If there are no leaks, the condition at the bottom is given by $\left[K(\psi)\left(\dfrac{\partial\psi}{\partial z}+1\right)\right]_{z=0}=0$ otherwise the condition changes to $\left[K(\psi)\left(\dfrac{\partial\psi}{\partial z}+1\right)\right]_{z=0}=\beta(t)$ where the function $\beta(t)$ is equal to the flux of the leak. It was shown in [6] that using the usual transformations

$$K(\psi)=K_s e^{\alpha\psi} \quad \text{and} \quad \theta=\theta_r+(\theta_s-\theta_r)e^{\alpha\psi} \qquad (2)$$

introducing the Kirchhoff transformation $\Phi(z,t)=K(\psi)/\alpha$ and approximating the function $\beta(t)$ with a piecewise linear function, i.e. $\beta(t)=\beta_0+\sum_{u=2}^{i}\beta_{u-1}(t_k-t_{k-1})+\beta_i(t-t_i)$, the transient water content distribution $\theta(z,t)$ and the flux potential $\Phi(z,t)$ are given by

$$\theta(z,t)=\theta_r+(\theta_s-\theta_r)\frac{\alpha}{K_s}\Phi(z,t) \qquad (3)$$

$$\Phi(z,t)=\Phi_s(z)+8D\mathrm{e}^{-\frac{\alpha z}{2}}\left\{\sum_{n=1}^{\infty}\frac{\left(\lambda_n^2+\frac{\alpha^2}{4}\right)sin(\lambda_n L)}{2\alpha+\alpha^2 L+4L\lambda_n^2}\bullet\right.$$

$$\left[\mathrm{e}^{\frac{\alpha L}{2}}sin(\lambda_n z)\int_0^t(q_0-q_1(\tau))\mathrm{e}^{-D\left(\lambda_n^2+\frac{\alpha^2}{4}\right)(t-\tau)}d\tau+\right. \qquad (4)$$

$$\left.\left.\mathrm{e}^{-D\left(\lambda_n^2+\frac{\alpha^2}{4}\right)t}\left(\lambda_n\cos(\lambda_n(L-z))+\frac{\alpha}{2}\sin(\lambda_n(L-z))\right)\sum_{u=1}^{i}\beta_u\xi_{un}\right]\right\}$$

$$0 \le z \le L$$

$$t_i \le t \le t_{i+1}$$

$$\Phi_s(z) = \beta_0 e^{-\alpha z} + \frac{q_0}{\alpha}\left(\mathrm{e}^{-\alpha z} - 1\right) + \int_0^L G(z,x) S(x) dx \qquad (5)$$

$$G(z,x) = \begin{cases} \dfrac{\mathrm{e}^{-\alpha z}}{z}\left(1 - \mathrm{e}^{-\alpha x}\right) & 0 \le x \le z \le L \\ \dfrac{1}{\alpha}\left(\mathrm{e}^{-\alpha z} - 1\right) & 0 \le z \le x \le L \end{cases}$$

$$\begin{cases} \xi_{un} = \dfrac{\left[(t_{u+1} - t_u) D\left(\lambda_n^2 + \dfrac{\alpha^2}{4}\right) - 1\right] \mathrm{e}^{-D\left(\lambda_n^2 + \frac{\alpha^2}{4}\right)(t - t_{u+1})} + \mathrm{e}^{-D\left(\lambda_n^2 + \frac{\alpha^2}{4}\right)(t - t_u)}}{\left[D\left(\lambda_n^2 + \dfrac{\alpha^2}{4}\right)\right]^2} \\ \xi_{in} = \dfrac{\left[(t - t_i) D\left(\lambda_n^2 + \dfrac{\alpha^2}{4}\right) - 1\right] + \mathrm{e}^{-D\left(\lambda_n^2 + \frac{\alpha^2}{4}\right)(t - t_i)}}{\left[D\left(\lambda_n^2 + \dfrac{\alpha^2}{4}\right)\right]^2} \end{cases} \qquad u<i$$

where λ_n is the n-th solution of the equation $sin(\lambda_n L) + \frac{2\lambda_n}{\alpha} cos(\lambda_n L) = 0$ and the soil parameters $\alpha, K_s, \theta_s, \theta_r, D\left(D = \frac{K_s}{\alpha(\theta_s - \theta_r)}\right)$ are the soil pore-size distribution, the hydraulic conductivity, the water content at saturation, the residual water content, and the soil moisture diffusivity, respectively.

The number of intervals that can be considered to increase the accuracy of the approximation is limited by the necessity of preventing the problem from becoming ill-posed.

Suitable regularization techniques are discussed in [5].

This solution can be rewritten as

$$\theta(z,t) - \theta_r = (\theta_s - \theta_r)\frac{\alpha}{K_s}\left[F_0(z,t) + \beta_0 F_1(z) + \sum_{u=1}^{i} \beta_u F_{2u}(z,t)\right] \qquad (6)$$

where $F_0(z,t), F_1(z,t)$ and $F_{2u}(z,t)$ can be evaluated by comparison or simplifying the notation

$y(t_k) \cong \sum_{u=0}^{i} \beta_u r_{uk}$ where $\cong$ means equality in the sense of least squares, y are *N* measured values of $\theta(L,t_k) - \theta_r - F_0(L,t_k)$ and $r_{uk} = \{F_1, F_{2u}(t_k)\}$.

Rewriting the flux at the bottom $\left.\frac{\partial\Phi(z,t)}{\partial z} + \alpha\Phi\right|_{z=0}$ as

$\left.\frac{\partial\Phi(z,t_k)}{\partial z} + \alpha\Phi(z,t_k)\right|_{z=0} = c(t_k) + \sum_{u=0}^{i} \hat{\beta}_u g_{uk}$, computing $c(t_k)$ and g_{uk} using the values of

$F_0(0,t), F_1(0,t), F_{2u}(0,t), \left.\frac{\partial F_0(z,t)}{\partial z}\right|_{z=0}, \left.\frac{dF_1(z)}{dz}\right|_{z=0}, \left.\frac{\partial F_{2u}(z,t)}{\partial z}\right|_{z=0}$ (see [6] for computational details), the hypothesis of absence of leaks at times t_k (i.e. $c(t_k)+\sum_{u=0}^{i}\hat{\beta}_u g_{uk}=0$) can be estimated using the statistic $\frac{\left(g_k^T\hat{\beta}+c_k\right)^2}{\hat{\sigma}^2\left[g^T\left(r^Tr\right)^{-1}g\right]}$ which is known to follow an *F*-distribution with 1 and *N-i-1* degrees of freedom.

The absence of leaks at all the measured times N is given by the condition $\sum_{k=1}^{N}\left(c(t_k)+\sum_{u=0}^{i}\hat{\beta}_u g_{uk}\right)=\sum_{k=1}^{N}c(t_k)+\sum_{k=1}^{N}\left(\sum_{u=0}^{i}\hat{\beta}_u g_{uk}\right)=\sum_{k=1}^{N}c(t_k)+\sum_{u=0}^{i}\hat{\beta}_u\sum_{k=1}^{N}g_{uk}=C+\sum_{u=0}^{i}\hat{\beta}_u G_u=0$ due to the fluxes being zero or negative and consequently by the statistic

$$\frac{\left(G^T\hat{\beta}+C\right)^2}{\hat{\sigma}^2\left[G^T\left(r^Tr\right)^{-1}G\right]} \qquad (7)$$

Thus, the calculated value of the statistic can provide the required probability for the absence of leaks at the bottom of the landfill.

Sensitivity analysis

While the analysis of experimental errors carried out in [5] shows that the significance of the test is roughly consistent with the accuracy of present day instrumentation, the influence of the parameters π contained in the model (i.e. $\pi=\{L,\alpha,K_s,\theta_s,\theta_r,q_0\}$) must also be taken into account to verify the reliability of the test.

To this purpose, the range of values of F corresponding to the uncertainties of the parameters is evaluated by computing $\frac{\partial F}{\partial \pi_i}\Delta\pi_i$, where $\Delta\pi_i$ is the range of possible values of parameter i , i.e.

$$\frac{\Delta\pi_i}{\hat{\sigma}^2}\frac{\partial}{\partial\pi_i}\frac{\left(G^T\hat{\beta}+C\right)^2}{\left[G^T\left(r^Tr\right)^{-1}G\right]} \qquad (8)$$

The following steps have been employed to evaluate (8).

(a) Compute the derivatives of the matrices by differentiating each element with respect to the parameters. Indeed, the differentiation of a matrix with respect to a scalar is given by a matrix whose elements are the derivatives of the elements of the original matrix

(b) Compute $\frac{\partial r_{kj}}{\partial\pi_i}, \frac{\partial C_{kj}}{\partial\pi_i}$ and $\frac{\partial G_{kj}}{\partial\pi_i}$ by direct differentiation of $F_0(0,t), F_1(0,t), F_{2u}(0,t)$, $\left.\frac{\partial F_0(z,t)}{\partial z}\right|_{z=0}, \left.\frac{dF_1(z)}{dz}\right|_{z=0}, \left.\frac{\partial F_{2u}(z,t)}{\partial z}\right|_{z=0}$ with respect to parameter *i*. Derivatives can be obtained using regular differentiation rules or automated differentiation [7] to get numerical values. The latter method has been used in this paper.

(c) Compute the derivatives of the product of two matrices using the same rule as for the product of two scalars keeping the multiplication order, i.e. $\frac{\partial}{\partial \pi_i}(AB)=\frac{\partial A}{\partial \pi_i}B+A\frac{\partial B}{\partial \pi_i}$

(d) Compute the derivatives of the inverse of a matrix using the formula $\frac{\partial A^{-1}}{\partial \pi_i}=-A^{-1}\frac{\partial A}{\partial \pi_i}A^{-1}$

This procedure has been applied to all parameters and its validity has been proven the automated differentiation technique provided by the powerful ADIFOR algorithm to evaluate the partial derivatives with respect to the parameter π_i

$$\frac{\partial \mathrm{F}_0(0,t)}{\partial \pi_i},\frac{\partial \mathrm{F}_1(0,t)}{\partial \pi_i},\frac{\partial \mathrm{F}_{2u}(0,t)}{\partial \pi_i},\left.\frac{\partial^2 \mathrm{F}_0(z,t)}{\partial z \partial \pi_i}\right|_{z=0},\left.\frac{\partial^2 \mathrm{F}_1(z)}{\partial z \partial \pi_i}\right|_{z=0},\left.\frac{\partial^2 \mathrm{F}_{2u}(z,t)}{\partial z \partial \pi_i}\right|_{z=0}.$$

Conclusions

The sensitivity analysis outlined in this paper serves a threefold purpose:

1) it might help select the soil with the most convenient parameters $\alpha, K_s, \theta_s, \theta_r$ as detected by the algorithm.
2) it might contribute to the optimal value of the landfill depth L in the design phase.
3) it provides useful insight about when to carry out an experimental campaign depending on intensity and duration of rainfall.

Thus, the availability of the overall algorithm and the inclusion of the relevant software in soil moisture meters can be regarded as a powerful tool in landfill monitoring.

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