Theoretical analysis of steady-state biooxydation under limitation and inhibition by substrate

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A stationary problem of the oxidative action of a representative biofilm is formulated taking into account the nonlinear kinetics of substrate utilization, which reflects its limitation and inhibition. Two characteristic situations are sequentially considered - high initial content of the substrate, relatively low oxidation rate (no inert zone); low substrate content, high oxidation rate (inert zone exists). The exact solution of the mathematical problem are obtained and compared. A number of special cases are considered. The subject of quantitative analysis was the consequences of biooxidation for different parts of the biofilm and its productivity.

Keywords: biofilm, substrate, biooxidation, utilization, limitation, inhibition, solution, modeling

Introduction

In porous media, the development and vital activity of microorganisms attached to the surface of the solid phase, as a rule, occurs under unfavorable conditions due to tightness of the microbial population, insufficient levels of easily decomposable organic substances, oxygen and biogenic elements, the presence of toxins, temperature conditions, etc [1, 2]. Functioning of biofilms is first limited by the consumption and composition of the substrate. Even with a small amount of dissolved organic substance in waste and natural waters, biofilms are not able to use it with maximum utility due to their physiological and structural features. At the same time, an inhibitory effect is also possible at a high concentration, which forces to modify the nonlinear kinetics with limiting saturation of the Monod (Michaelis-Menten, Langmuir, etc) type or other complex kinetic equations. To formally take into account the specific degradation of the substrate in a particular biofilm, it is necessary to set the function of intrabiofilm utilization $u(s)$ in a special mathematical model, which authentically reflects a relationship between the rate of the process and the concentration of the substrate.

Statement and exact solution of the mathematical problem

When formulating the stationary problem of the oxidative action of a biofilm, it is customary to consider a system of assumptions, namely, to assume that it is homogeneous, its surface is flat, and a liquid film adjoins it. The original mathematical problem proposes an equation for the diffusion transfer of a substrate

$$D_e \frac{d^2 s}{dx^2} = \frac{\mu_m B}{Y_{BS}} u(s) = \lambda_B u(s); \quad (1)$$

boundary conditions at the internal and external $(x = l_f)$ surfaces of the biofilm

$$x = 0, \quad \frac{ds}{dx} = 0; \qquad x = l_f, \; D_e \frac{ds}{dx} = k_{\mathrm{L}}(S - s). \quad (2)$$

Here, s is the mass concentration of the substrate inside the biofilm; D_e is the diffusion coefficient; λ_B is the reduced coefficient of the maximum biooxidation rate; μ_m is the maximum specific rate of the biomass growth; Y_{BS} is the economic coefficient of substrate transformation into biomass; l_f is the (equivalent) biofilm thickness; k_{L} is the mass transfer coefficient through the liquid film adjacent to the biofilm; S is the mass concentration of the substrate outside both films;

origin of the coordinate axis is at the inner boundary of the biofilm; the form of the function $u(s)$ will be specified below. The desired concentration s is found in the form of the following inverse integral function

$$x = l_f - \sqrt{\frac{D_e}{2\lambda_B}} \int_s^{s_f} \frac{d\eta}{\sqrt{U(\eta, s_g)}}. \quad (3)$$

where $U(s, s_g) = \int_{s_g}^{s} u(\zeta) d\zeta$, s_g is the mass concentration of the substrate at the inner boundary. Eq. (3) contains unknown parameters s_g, s_f (mass concentration of the substrate at the outer surface of the biofilm). To determine them using conditions (2), a system of equations is compiled

$$U(s_f, s_g) = \frac{k_L^2}{2\lambda_B D_e}(S - s_f)^2, \quad (4)$$

$$l_f = \sqrt{\frac{D_e}{2\lambda_B}} \int_{s_g}^{s_f} \frac{d\eta}{\sqrt{U(\eta, s_g)}}. \quad (5)$$

Thus, the required parameters are in fact implicit functions of the concentration S. To generalize the theoretical analysis, dimensionless variables and parameters are introduced: $\bar{s} = s/s_0$, $\bar{s}_{f,g} = s_{f,g}/s_0$, $\bar{S} = S/s_0$, $\bar{x} = x/l_0$, $\bar{l}_f = l_f/l_0$, $\bar{\lambda}_B = \lambda_B l_0^2/(D_e s_0)$,

s_0 is the initial concentration of the substrate (at the entrance to the bioreactor-filter), l_0 is the length scale, for example, the radius of the granule (grain) R_g .Then the solution of problem (1), (2) is represented by such a system of dimensionless equations

$$U(\bar{s}_f, \bar{s}_g) = \frac{\bar{k}_L^2}{2\bar{\lambda}_B}(\bar{S} - \bar{s}_f)^2, \quad (6)$$

$$\bar{l}_f = \frac{1}{2\sqrt{\bar{\lambda}_B}} \int_{\bar{s}_g}^{\bar{s}_f} \frac{d\eta}{\sqrt{U(\eta, \bar{s}_g)}}, \quad (7)$$

$$\bar{x} = \bar{l}_f - \frac{1}{\sqrt{2\bar{\lambda}_B}} \int_{\bar{s}}^{\bar{s}_f} \frac{d\eta}{\sqrt{U(\eta, \bar{s}_g)}}. \quad (8)$$

It is possible to express the integral function $U(\bar{s}, \bar{s}_g)$ in terms of elementary functions for some forms of function $u(s)$ proposed in the literature. Thus, in the case of the kinetics of Haldane, which takes into account the inhibitory effect [3],

$$u(s) = \frac{s^2}{s^2 + \bar{K}_l s + K_i}, \qquad u(\bar{s}) = \frac{\bar{s}^2}{\bar{s}^2 + \bar{K}_l \bar{s} + \bar{K}_{ii}}, \quad (9)$$

where $\bar{K}_s$ is an analogue of the half-saturation constant; K_i is the inhibition coefficient; $\bar{K}_{s,i} = K_{s,i}/s_0$, the following representation is valid

$$U(\bar{s},\bar{s}_g)=\int_{\bar{s}_g}^{\bar{s}}u(\zeta)d\zeta=$$

$$=\frac{\bar{K}_i}{2}\left[\ln\frac{\bar{s}^2+\bar{K}_i\bar{s}+\bar{K}_s\bar{K}_i}{\bar{s}_g^2+\bar{K}_i\bar{s}_g+\bar{K}_s\bar{K}_i}-\frac{1}{\sqrt{\Delta_K}}\ln\left(\frac{2\bar{s}+\bar{K}_i-\sqrt{\Delta_K}}{2\bar{s}+\bar{K}_i+\sqrt{\Delta_K}}\;\;\frac{2\bar{s}_g+\bar{K}_i+\sqrt{\Delta_K}}{2\bar{s}_g+\bar{K}_i-\sqrt{\Delta_K}}\right)\right],$$

where $\Delta_k=\bar{K}_i^2-4\bar{K}_s\bar{K}_i$. If there is a nonlinear kinetics of the Mono type $(K_i=0)$, then the system of Eqs. (6) - (8) is written in such a final form [4]

$$\bar{s}_f-\bar{s}_g-\bar{K}_s\ln\frac{\bar{s}_f+\bar{K}_s}{\bar{s}_g+\bar{K}_s}=\frac{\bar{k}_L^2}{2\bar{\lambda}_B}(\bar{S}-\bar{s}_f)^2,\quad(10)$$

$$\frac{1}{\sqrt{2\bar{\lambda}_B}}\int_{\bar{s}_g}^{\bar{s}_f}\frac{d\zeta}{\sqrt{\zeta-\bar{s}_g-\bar{K}_s\ln\frac{\zeta+\bar{K}_s}{\bar{s}_g+\bar{K}_s}}}=\bar{l}_f,\quad(11)$$

$$\bar{x}=\bar{l}_f-\frac{1}{\sqrt{2\bar{\lambda}_B}}\int_{\bar{s}}^{\bar{s}_f}\frac{d\zeta}{\sqrt{\zeta-\bar{s}_g-\bar{K}_s\ln\frac{\zeta+\bar{K}_s}{\bar{s}_g+\bar{K}_s}}}.\quad(12)$$

The limiting cases of the indicated kinetics, which correspond to low $(K_s>>s)$ and high $(s>>K_s)$ substrate content are of interest from the methodological point of view. In the first case

$$\bar{\lambda}_B u(\bar{s})=\tilde{\lambda}_B\bar{s},\qquad U(\bar{s},\bar{s}_g)=0.5\bar{\lambda}_B(\bar{s}^2-\bar{s}_g^2),$$

where $\tilde{\lambda}_B=\bar{\lambda}_B/\bar{K}_s$. As a result, we receive

$$\bar{s}(\bar{x})=\frac{\bar{k}_L\bar{S}\left(e^{\sqrt{\tilde{\lambda}_B}\bar{x}}+e^{-\sqrt{\tilde{\lambda}_B}\bar{x}}\right)}{\left(\sqrt{\tilde{\lambda}_B}+\bar{k}_L\right)e^{\sqrt{\tilde{\lambda}_B}\bar{l}_f}-\left(\sqrt{\tilde{\lambda}_B}-\bar{k}_L\right)e^{-\sqrt{\tilde{\lambda}_B}\bar{l}_f}}.\quad(13)$$

It follows from Eq. (13) that at any values $\bar{S}>0$ the substrate reaches the inner surface and therefore the entire mass of the biofilm will be active $(s_g>0)$.

A different picture emerges at the maximum rate of substrate utilization $(u=1)$. In such a situation, it is useful to further substantiate the assumption of biofilm homogeneity by generalizing a simple initial model, additionally taking into account the actual biofilm heterogeneity. Indeed, according to [5], the biomass is unevenly distributed over the thickness, so that its density, and at the same time the coefficient D_e, are functions of x. Then the original equation (1) after an appropriate adjustment is reduced to the following form

$$\frac{d}{dx}\left[D(x)\frac{ds}{dx}\right]=\lambda_B(x)=\frac{\mu_m}{Y_{BS}}\rho_B(x),\quad(14)$$

where μ_m is the maximum specific biomass growth rate; Y_{BS} is the economic coefficient of substrate transformation into biomass; ρ_B is the biomass density. After the introduction of dimensionless variables $\bar{D}(\bar{x})=D(x)/D_e$, $\bar{\rho}_B(\bar{x})=\rho_B(x)/\rho_{B0}$ Eq. (14) is transformed as follows

$$\frac{d}{d\bar{x}}\left[\overline{D}(\bar{x})\frac{d\bar{s}}{d\bar{x}}\right]=\lambda_{B0}\overline{\rho}_B(x),\ (15)$$

where $\lambda_{B0}=\mu_m\rho_{B0}l_0^2/(Y_{sB}D_e s_0)$.

Biofilm material can be in both an active and inert state, and deprived of nutrition, it cannot, in principle, persist for a long time. Indeed, from the kinetic equation for inert biomass with concentration B_i

$$\frac{\partial B_i}{\partial t}=-k_d(B_i)B$$

it follows that the inert biomass will monotonically decrease with time (with the functional coefficient of the biomass loss rate $k_d=const$ - according to an exponential law) and, at any initial value, asymptotically $(t\to\infty)$ tend to 0. By the way, with a sharp decrease in the concentration of the substrate at the inlet of the operating filter, the inert zone either appears or increases abruptly, but then gradually becomes thinner again due to the prevalence of its costs for autoxidation. However, with the constant replenishment of inert biomass reserves due to the active biomass, the corresponding zone can exist for a long time.

In view of the possible formation of a zone with inert biomass, two typical situations should be considered sequentially. The first situation formed at a high initial content of the substrate and a relatively low rate of its oxidation. Then conditions (2) must be satisfied, and solution (15) is expressed by the general dependence

$$\bar{s}(\bar{x})=\overline{S}+\frac{\lambda_{B0}}{\overline{k}_L}\int_0^{\bar{l}_f}\overline{\rho}_B(\eta)d\eta+\lambda_{B0}\int_{\bar{l}_f}^{\bar{x}}\frac{1}{\overline{D}(\eta)}\int_0^{\eta}\overline{\rho}_B(\zeta)d\zeta d\eta.\ (16)$$

Hence, we can conclude that with correct averaging $\overline{\rho}_B$ and $\overline{D}=1$, the accuracy of calculations $\bar{s}_f$ for any form of dependence $\overline{\rho}_B(\bar{x})$ does not decrease. Obviously, the entire biofilm will be active under the condition $\bar{s}(0)\ge 0$. A state can be considered as critical if $\bar{s}(0)=0$. Then any deterioration in living conditions, formally expressed in a hypothetical increase in $\bar{l}_f$, $\overline{k}_L$, λ_{B0}, $\overline{\rho}_B$ or a decrease in $\overline{S}$, $\overline{D}$, will lead to the appearance of an inert zone. The relationship between these parameters, corresponding to the critical state, is described by the equation

$$\lambda_{B0}\int_0^{\bar{l}_f}\frac{1}{\overline{D}(\eta)}\int_0^{\eta}\overline{\rho}_B(\zeta)d\zeta\,d\eta+\frac{\lambda_{B0}}{\overline{k}_L}\int_0^{\bar{l}_f}\overline{\rho}_B(\eta)d\eta=\overline{S}.\ (17)$$

If the inert zone exists, so that $\bar{l}_*>0$ (l_* is the coordinate of the boundary between the active and the indicated zones), then instead of the first condition (2) it is necessary to specify a pair of boundary conditions

$$\bar{x}=\bar{l}_*,\qquad \bar{s}=0;\qquad \frac{d\bar{s}}{d\bar{x}}=0.\ (18)$$

Then the solution of problem (15), (18) is represented in the following form .

$$\bar{s}(\bar{x})=\overline{S}-\frac{\lambda_{B0}}{\overline{k}_L}\int_{\bar{l}_*}^{\bar{l}_f}\overline{\rho}_B(\eta)d\eta+\overline{\lambda}_{B0}\int_{\bar{l}_f}^{\bar{x}}\frac{1}{\overline{D}(\eta)}\int_{\bar{l}_*}^{\eta}\overline{\rho}_B(\zeta)d\zeta\,d\eta.\ (19)$$

The value $\bar{l}_*$ is not known in advance and must be found from the equation

$$\lambda_{B0}\int_{\bar{l}_*}^{\bar{l}_f}\frac{1}{\overline{D}(\eta)}\int_{\bar{l}_*}^{\eta}\bar{\rho}_B(\zeta)d\zeta\,d\eta+\frac{\lambda_{B0}}{\bar{k}_L}\int_{\bar{l}_*}^{\bar{l}_f}\bar{\rho}_B(\eta)d\eta=\bar{S}\ . \quad (20)$$

According to [6] for functions $\overline{D}(\bar{x})$, $\rho_B(\bar{x})$, the linear form is quite acceptable, namely,

$$\overline{D}(\bar{x})=a_D\bar{x}+b_D,\qquad \rho_B(\bar{x})=a_\rho\bar{x}+b_\rho\ , \quad (21)$$

where $a_{D,\rho}$, $b_{D,\rho}$ are empirical constants. Taking into account Eq. (21) and as a result of integration, dependence (16) is represented as follows

$$\bar{s}(\bar{x})=\bar{S}+\frac{\lambda_{B0}}{2}\left[\frac{a_\rho}{2a_D}\left(\bar{x}^2-\bar{l}_f^2\right)-\frac{a_\rho\bar{l}_f^2+2b_\rho\bar{l}_f}{\bar{k}_L}+\frac{2a_Db_\rho-a_\rho b_D}{a_D^2}\left(\bar{x}-\bar{l}_f\right)-\right.$$
$$\left.-\frac{2a_Db_Db_\rho-a_\rho b_D^2}{a_D^3}\ln\frac{a_D\bar{x}+b_D}{a_D\bar{l}_f+b_D}\right]. \quad (22)$$

If we neglect changes in the diffusion coefficient $\left(\overline{D}=1\right)$, then (22) will be simplified so that

$$\bar{s}(\bar{x})=\bar{S}+\frac{\lambda_{B0}}{2}\left(\frac{a_\rho}{3}\bar{x}^3+b_\rho\bar{x}^2-\frac{a_\rho\bar{l}_f^2+2b_\rho\bar{l}_f}{\bar{k}_L}-\frac{a_\rho\bar{l}_f^3}{3}-b_\rho\bar{l}_f^2\right). \quad (23)$$

Finally, in the simplest case of unlimited oxidation in a biofilm with a uniformly distributed biomass $\left(\overline{D}=\bar{\rho}_B=1\right)$ and in the absence of an inert zone, changes in the relative concentration of the substrate are described by the dependence

$$\bar{s}(\bar{x})=\bar{S}-\frac{\bar{\lambda}_B\bar{l}_f}{\bar{k}_L}+\frac{\bar{\lambda}_B}{2}\left(\bar{x}^2-\bar{l}_f^2\right). \quad (24)$$

The specified zone $\left(\bar{l}_*\le\bar{x}\le\bar{l}_f\right)$ will be formed when the condition is met

$$\bar{S}<\frac{\bar{\lambda}_B\bar{l}_f}{2\bar{k}_L}\left(\bar{k}_L\bar{l}_f+2\right). \quad (25)$$

Then the dependence similar to (24) will be

$$\bar{s}(\bar{x})=\frac{\bar{\lambda}_B}{2}\left(\bar{x}-\bar{l}_f+\frac{1}{\bar{k}_L}+\sqrt{\frac{1}{\bar{k}_L^2}+\frac{2\bar{S}}{\bar{\lambda}_B}}\right)^2, \quad (26)$$

and the relative coordinate of the boundary between the active and inert zones should be calculated by the formula

$$\bar{l}_*=\bar{l}_f+\frac{1}{\bar{k}_L}-\sqrt{\frac{1}{\bar{k}_L^2}+\frac{2\bar{S}}{\bar{\lambda}_B}}\ . \quad (27)$$

Directly, the content of dissolved organic compounds in the working volume of bioreactors with a fixed biocenosis decreases due to their inflow into biofilms, which together form a biological phase. And it is natural that the productivity of such bioreactors is directly related to the diffusion transfer of the substrate into a separate biofilm, as well as their total amount. Therefore, when

modeling the operation of devices with a stationary biological phase, the key role is played by the determination of the transport of the flow of dissolved organic matter into a representative biofilm i_f, which can be expressed in two ways, namely,

$$i_f = 4\pi\left(R_g + l_f\right)^2 \cdot k_L\left(S - s_g\right) = 4\pi\left(R_g + l_f\right)^2 D_e \left.\frac{ds}{dx}\right|_{x=\lg} .$$

Thus, if we accept $l_0 = R_g$, then the relative value $\bar{i}_f$ will be

$$\bar{i}_f = \frac{i_f}{4\pi R_g D_e s_0} = \bar{k}_L\left(1 + \bar{l}_f\right)^2\left(\bar{S} - \bar{s}_g\right) = \left(1 + \bar{l}_f\right)^2 \left.\frac{d\bar{s}}{d\bar{x}}\right|_{\bar{x}=\bar{l}_f} . \quad (28)$$

It is appropriate to note here that, due to the presence of the factor $\left(1 + \bar{l}_f\right)^2$ in expressions (28), it is possible to avoid additional errors in the calculations of the action of the biofilm first, and then of biofiltration due to the neglect of its curvature. These errors are due to the actual difference between the areas of the inner and outer surfaces of the biofilm and are measured by the following relationships

$$\frac{\left(R_g + l_f\right)^2 - R_g^2}{\left(R_g + l_f\right)^2} = \frac{\bar{l}_f\left(1 + 2\bar{l}_f\right)}{\left(1 + \bar{l}_f\right)^2} \approx \bar{l}_f . \quad (29)$$

Therefore, to establish the value $\bar{i}_f$ at a given concentration, it is enough to first find the value $\bar{s}_f$ based on the system, for example (10), (11) or, in extreme cases, using formulas (13), (16), (19). At an unknown value $\bar{S}$, it is necessary to formulate and solve, in essence, a problem external to the biofilm. The relevant research materials by analytical methods will be presented in the next section.

Calculation of examples and discussion of results

The exact solution obtained above for the stationary problem of substrate oxidation in a separate biofilm and its special cases are illustrated by calculations of relative microcharacteristics, which are of particular interest for mathematicalmodeling of biofiltration. The subject of quantitative analysis was the consequences of biooxidation in general for the peripheral part of the biofilm $\left(\bar{s}_g\right)$, as well as its productivity $\left(\bar{i}_f\right)$. Taking into account the direction of theoretical studies of the oxidative action of an individual biofilm and the peculiarities of the functioning of a set of biofilms in a bioreactor-filter, the concentration of the substrate in the surrounding solution was used as an argument in determining the desired functional values. At the same time, it $\left(\bar{S}\right)$ continuously changed within the maximum possible range - from 0 to 1. In addition, the value of the parameter $\bar{K}_s$ (0.5) was fixed, and the values of the other model parameters $\left(\bar{\lambda}_B,\ \bar{k}_L,\ \bar{l}_f\right)$ were varied with the intention to more widely cover the variety of technological conditions when filtering water with significantly different content of easily decomposable organic substances.

In the first place, strict Eqs. (10) and (11) were used for calculations, as well as, in addition to them, simple formulas (13), (24), (28), corresponding to the limiting cases of nonlinear kinetics (Mono).

First of all, the concentration of the substrate on the inner surface of the biofilm was determined as the hypothetical increase in water pollution outside it (Fig. 1). The values $\bar{s}_g$ were calculated, firstly, for strictly limited decomposition of the substrate and three values of the coefficient $\bar{\lambda}_B$ (curves 1–3), and secondly, for linear kinetics, unlimited decomposition and $\bar{\lambda}_B = 50$

. It has been established that with both methods of limiting the rate of degradation of dissolved organic substance, even at arbitrarily small values $\bar{S}$, some of its amount is retained in the entire biofilm. At a constant rate of substrate biooxidation, according to (25), it is sufficient to reduce the value $\bar{S}$ to 0.55 for it to be completely utilized inside the biofilm. Naturally, with an even greater decrease of $\bar{S}$, an inert zone will necessarily form near the substrate, so that $\bar{l}_* < 1$. The limiting effect is easy to visually assess by comparing graphs 1, 2, 5, corresponding to the same value $\bar{\lambda}_B$.

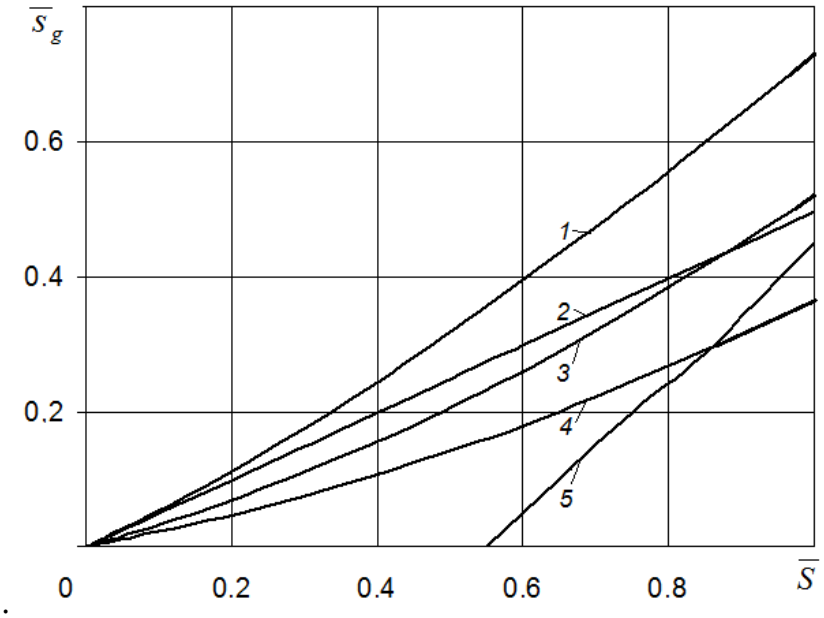

Fig.1. Dependence $\bar{s}_g(\bar{S})$:

1, 2, 5 – $\bar{\lambda}_B = 50$; 3 – $\bar{\lambda}_B = 100$; 4 – $\bar{\lambda}_B = 150$; 1, 3, 4 – nonlinear kinetics (Mono); 2 – linear; 5 – $u = 1$

The actual oxidative capacity of a biofilm is characterized by the rate of substrate inflow into it. Its consumption inside the biofilm $\bar{i}_f$ significantly depends on their physical and biochemical properties and was determined in all cases considered by formula (28). The input concentration $\bar{s}_f$ was preliminarily determined using the same calculation procedure and with the same initial data as $\bar{s}_g$. In two series of calculations, the coefficient $\bar{\lambda}_B$ and thickness $\bar{l}_f$ alternately changed by an order of magnitude and by 2...4 times respectively. The data of calculating dependence $\bar{i}_f(\bar{S})$ for three forms of biooxidation kinetics are presented in the form of two sets of graphs in Figs. 2, 3. With an increase in the initial water pollution, the slope of all graphs gradually decreases. Thus, on the one hand, it is advisable to maintain a high content of the substrate outside the fixed biocenosis for its productive activity. But, on the other hand, the quality of water treatment is of decisive importance for the management of a bioreactor. The calculated flow $\bar{i}_f$ shows high sensitivity with respect to each of the three variable parameters. To a greater extent, it responds to changes in the relative thickness of the active biofilm. Thus, a formal increase of $\bar{l}_f$ from 0.05 to 0.2 leads to an increase in the substrate inflow from the outside by 2–2.5 times. At a stable rate of utilization of organic substance $(u = 1)$, much more of it enters the biofilm, and the flow rate $\bar{i}_f$ reaches its maximum value (in the absence of an inert zone, it is equal to $\bar{\lambda}_B \bar{l}_f$). Then, in particular, at $\bar{\lambda}_B = 50$, $\bar{l}_f = 0.1$ and also due to $\bar{k}_L = 25$ и $\bar{S} = 1$ a significant limitation of the substrate degradation rate,

the value $\bar{i}_f$ is reduced by 1.6-2.4 times. In the case of linear kinetics with a coefficient $\bar{\lambda}_B/\bar{k}_l$, in comparison with the kinetics of the Mono type, a serious overestimation of the rates of biooxidation and volumes of treated water is possible.

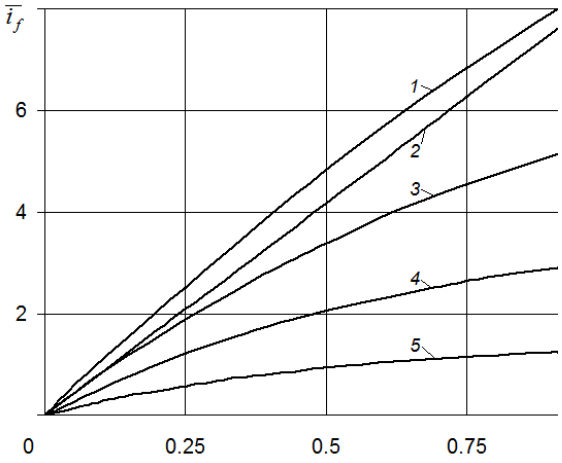

Fig.2. Dependence $\bar{i}_f(\bar{S})$:
$1 - \bar{\lambda}_B = 200,\ 2 - \bar{\lambda}_B = 100,\ 3 - \bar{\lambda}_B = 50,$
$4 - \bar{\lambda}_B = 20$

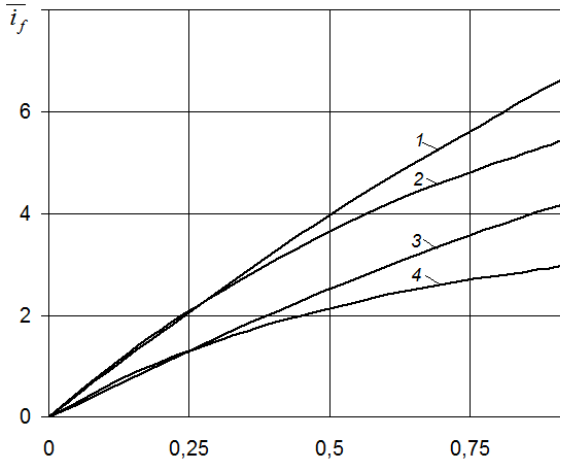

Fig 3. Dependence $\bar{i}_f(\bar{S})$:
$1 - \bar{l}_f = 0.2;\ 2, 3 - \bar{l}_f = 0.1;\ 4 - \bar{l}_f = 0.05;$
$1, 4 - \bar{k}_L = 25;\ 2 - \bar{k}_L = 40;\ 3 - \bar{k}_L = 10$

Conclusions

Summarizing the above theoretical studies, it follows highlight:

- the validity of the set of assumptions adopted when choosing the basic mathematical model of the oxidative action of a separate biofilm and, as a result, its adequacy in general to the operating conditions of bioreactors with fixed biota,

- the rigor of the derived calculation equations and dependencies, which allows them to be used in the subsequent study of the patterns of biofiltration, as well as reference expressions in the development and evaluation of the reliability of approximate analogues that are convenient for technological and constructive analysis.

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