Some aspects determination of heat exchange characteristics during the flow of non-Newtonian fluids

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***Abstract* – The problem of heat transfer of non-Newtonian fluids in the channels of chemical-technological equipment is considered. The obtained expressions, when carrying out engineering calculations, allow us to calculate the corresponding heat transfer and heat transfer coefficients during the flow of non-Newtonian fluids in the channels and with the environment.**

Keywords – non-Newtonian fluid, flow, heat transfer, pipe, channel, Nusselt number.

Introduction

Heat exchange plays an important role in the processes of the chemical and food industries. A detailed study of the structure of heat flow allows a high level of organization of technological processes. It is known that most liquids used in the production of chemical and food products have an abnormal flow character, so studying the process of heat exchange of non-Newtonian fluids is very relevant.

In this work the authors study the calculation of the coefficients of heat transfer during the flow of non-Newtonian fluid in a pipe or channel of chemical and technological equipment. From the analysis of the technical literature, we can conclude that among the variety of non-Newtonian fluids, the most common are three classes: Bingam fluids, generalized displaced fluids and power fluids. Under the term "generalized-displaced fluids" we mean liquids whose viscosity depends on the shear rate in an arbitrary manner. A special case of such fluids is power fluid [2].

Results

This paper deals with stabilized flows of non-Newtonian fluids in a hydrodynamic sense and with destabilized temperature transfer with respect to the thermal boundary layer. The latter condition means that the Péclet number is much greater than one. The convective temperature transfer equation is used to calculate the heat transfer coefficients [3]. The heat transfer is affected by the velocity component, which can be both tangent and normal relative to the heat transfer surface; in straight channels and pipes with a stable flow, the normal velocity component is absent. A tangential velocity component can have two components - along and across the longitudinal axis of a pipe or channel. In this case, the tangent velocity component is the vector sum of these components, and it is this sum that determines the heat transfer coefficient.

The equation of convective temperature transfer is written as follows:

$$\upsilon_x \frac{\partial T}{\partial x} + \upsilon_y \frac{\partial T}{\partial y} = \chi \frac{\partial^2 T}{\partial y^2}, \quad \chi = \frac{\lambda}{\rho c_p} \qquad (1)$$

where υ_x and υ_y – tangent and normal components of non-Newtonian fluid velocity vector, m/s; T – absolute temperature of the liquid, K; χ – temperature conductivity of the liquid, m^2/s; λ – thermal conductivity of the liquid, W/m·K; ρ – fluid density, kg/ m^3; c_p – heat capacity of the liquid, J/kg·K. Equation (1) is written in the approximation of the thermal boundary layer so that only the transverse derivative of the variable y is preserved in its right side. The x coordinate is considered to be aligned along the tangent component of the velocity of the fluid (in the case of purely longitudinal flow, the tangent component is directed along the axis of the pipe or channel).

Consider $\upsilon_y \equiv 0$, given the fact that there is a thermal boundary layer and the heat flux near the solid surface depends on the behavior of the velocity field only near that surface. The second and first order decompositions for Bingam and non-Bingam fluids, respectively, should be used according to the small distance to the solid surface. If this distance is denoted as, the $\tilde{y}$ following expression should be used for Bingam and non-Bingam fluids near the boundaries of sections of the flow (channel pipe walls):

$$\upsilon_x = w_\Gamma + \frac{\partial \upsilon_x}{\partial \tilde{y}}\tilde{y}, \qquad (2)$$

while for Bingam fluid near a solid core, the following expression should be used:

$$\upsilon_x = \upsilon_k + \frac{\partial^2 \upsilon_x}{\partial \tilde{y}^2}\tilde{y}^2, \qquad (3)$$

where w_Γ - velocity of the flow near wall, m/s; υ_k - velocity of the solid core, m/s. In (3) addition, proportional to $\tilde{y}$ is absent because the second invariant of the deformation rate tensor turns to zero [2].

Equations (1) from υ_x to (2) and (3) allows self-driving solutions with the help of substitutions of this following kind [4]:

$$\omega = \left(\frac{2}{\chi}\frac{\partial \upsilon_x}{\partial \tilde{y}}\right)^{1/3}\frac{\tilde{y}}{x^{1/3}} \text{ (for } \upsilon_x \text{ according to formula (2));}$$

$$(4)$$

$$\omega = \left(\frac{2}{\chi}\frac{\partial^2 \upsilon_x}{\partial \tilde{y}^2}\right)^{1/4}\frac{\tilde{y}}{x^{1/4}} \text{ (for } \upsilon_x \text{ according to formula (3))}$$

as see from (4), the heat flow density decreases along the direction of the tangent velocity $x^{-1/3}$ and $x^{-1/4}$ for first and second cases respectively. By entering the average value of the heat flow density at some length L and considering the standard definition of the Nusselt number for the latter, we obtain the following expressions:

$$Nu = \frac{2^{2/3}}{3}\frac{h}{L^{1/3}}\left(\frac{1}{\chi}\frac{\partial \upsilon_x}{\partial \tilde{y}}\right)^{1/3}, \quad Nu = \frac{8h}{15L^{1/4}}\left(\frac{1}{\chi}\frac{\partial \upsilon_x}{\partial \tilde{y}}\right)^{1/4}, \qquad (5)$$

in which h – half-width of the channel, pipe, m. Thus, it follows from (5) that the Nusselt numbers are determined by the derivatives of the velocity tangent to the wall on the walls of channels, pipes and at the boundaries of the solid core for the Bingam fluid.

In this paper we will consider the calculation of heat transfer coefficients for the flow of a power fluid. Due to the fact that a number of coolants is characterized by the viscosity of a power fluid [5]. Due to the fact that the main points of the calculations are exactly similar to those described for Bingam and generalized liquids [6], only the longitudinal flow of a power fluid in a flat channel is considered below. The expression for the velocity of the longitudinal flow of a power fluid with exponent is as follows:

$$\upsilon^{\pm} = \left| \frac{y - y^{*}}{\beta} \cdot \frac{dP}{dz} \right|^{\frac{n+2}{n+1}} \cdot \frac{n+1}{n+2} \frac{\beta}{dP/dz} - \left| \frac{h \mp y^{*}}{\beta} \frac{dP}{dz} \right|^{\frac{n+2}{n+1}} \cdot \frac{n+1}{n+2} \frac{\beta}{dP/dz} + w^{\pm},$$

$$y^{*} = \frac{w^{+} - w^{-}}{2\left(\frac{h}{\beta} \frac{dP}{dz} \right)^{\frac{1}{n+1}}}, \qquad \mu = \beta \left| \frac{d\upsilon^{\pm}}{dy} \right|^{n}. \quad (6)$$

The calculation of the derivative of (6) for the heat transfer coefficient results in the following result:

$$\left. \frac{\partial \upsilon^{\pm}}{\partial y} \right|_{y=\pm h} = \left(\frac{h \mp y^{*}}{\beta} \frac{dP}{dz} \right)^{\frac{1}{n+1}}. \quad (7)$$

The presented resultss indicate that the dependency of the Nusselt number on the pressure gradients (longitudinal and transverse) of the rheological and geometric characteristics of liquids and channels is nonlinear and very complex. In order to make this dependency simpler and clearer, you should write down the expressions for the Nusselt number for the simplest flow, that is, for the longitudinal flow in a flat channel. The more complex flows also have Nusselt numbers, but the numbers are more quantitative than the complicated ones.

The expressions for the Nusselt numbers below are written according to formulas (5) up to the trivial factors at velocity derivatives. For Bingam fluid, the Nusselt number is proportional to the following expression:

$$\mathrm{Nu} \sim \left\{ \frac{1}{\mu} \frac{dP}{d\zeta} \left[1 \mp \frac{\tau_0}{h \ldots} \ldots \frac{(w^{+} - w^{-}) \ldots}{\ldots} \right] \right\}^{\frac{1}{3}}$$ [illegible], (on the walls)

$$\mathrm{Nu} \sim \left(\frac{1}{\mu} \frac{dP}{d\zeta} \right)^{1/4}$$, (at the boundaries of the solid nucleus) (8)

From this expression it follows that the first two Nusselt numbers depend on three parameters: $(1/\mu)(dP/d\zeta)$; $\tau_0/(dP/dz)$; $(w^{+} - w^{-})/2h$, the last of which is the kinematic velocity of the displacement of the Couett flow of the Newtonian fluid.

For a generalized fluid, the correct expression for Nusselt numbers is the following:

$$\mathrm{Nu} \sim \left\{ \pm \frac{\alpha}{2\beta} \pm \left[\frac{\alpha^2}{4\beta^2} \pm \frac{1}{\beta} \frac{dP}{d\zeta} \cdot \left(1 \mp \frac{(w^{+} - w^{-})/2h}{\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{1}{\beta} \frac{dP}{d\zeta} \right)^{1/2}} \right) \right]^{1/2} \right\}^{1/3}, \quad (9)$$

From which it can be seen that the Nusselt number also depends on the following three parameters: $\alpha/2\beta$; $(1/\beta)(dP/d\zeta)$; $(w^{+}-w^{-})/2h$.

The power fluid has a Nusselt number proportional to the following expression:

$$\mathrm{Nu} \sim \left\{ \frac{1}{\beta}\frac{dP}{d\zeta}\left[1 \mp \frac{\left(w^{+}-w^{-}\right)/h}{\left(\frac{1}{\beta}\frac{dP}{d\zeta}\right)^{\frac{1}{n+1}}} \right] \right\}^{\frac{1}{3(n+1)}} \quad (10)$$

в яке входить два такі параметри: $(1/\beta)(dP/d\zeta)$ та $\left(w^{+}-w^{-}\right)/2h$.

Conclusions

Based on the above, we can draw the following conclusions. Nusselt numbers for Bingam and generalized fluids depend on three parameters for longitudinal flow in a flat channel. If a flow is more complex, that is, two or three-dimensional, then the number of parameters increases so that these parameters are generated by each velocity component of the multidimensional flow and form all possible combinations.

Subsequently, the value of Nusselt numbers allows us to calculate the corresponding coefficients of heat transfer and heat return between non-Newtonian fluids, pipes and channels, and the environment.

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